CHAPTER 2

Semiconductor Physics

Syllabus

- Qualitative ideas on formation of electron energy bands in solids
- Band-theory based classification of solids into insulators
- Semiconductors and conductors
- Intrinsic semiconductors
- Germanium and silicon
- Typical energyband diagram of an intrinsic semiconductor
- Fermi enrgy
- Doping and extrinsic semiconductors
- Dependence of Fermi energy on temperature and doping concentration
- Current conduction in semiconductors
- P-n junction diode
- Diode rectifier equation
- Transistor action
- Hall effect

2.1   Introduction

- From the point of view of engineering applications, it is essential to have the knowledge of the behaviour of materials as regards their magnetic, electric optical, dielectric, thermal, mechanical properties.

- Quantum mechanics plays a fundamental role in determining all these properties of materials.

- From the electrical point of view, the materials can be classified according to their resistivity range as

  o   Conductors

  o   Semiconductors

  o   Insulators

- Conductors have low resistivity (hence high conductivity) in the range 1.6×10^{-8} to 10^{-6} ohm-m.

- For insulators the range of resistivity is very high about 10^7 to 10^{16} ohm-m. Hence they are very poor conductors of electricity. Semiconductors are the materials whose resistivity (and hence the conductivity) lies between those of conductors and insulators, and has a range of about 10^{-4} to 10^6 ohm-m.

- In this chapter, we will be dealing with the semiconductors. The semiconductor devices are highly compact, efficient, more reliable, low power consuming, free from mechanical noise and are cheap. Hence they have replaced the vacuum tubes to a great extent.

- The properties of semiconductors can be understood on the basis of the band theory of solids. For this, we first consider the electron energy states of an isolated atom that can be described with the quantum numbers.

- In this chapter we also discuss the specific mechanisms by which current flows in a solid.

- In examining all these mechanisms we shall learn why some materials are good conductors of electric current, whereas others are poor conductors.

- We shall discuss how the conductivity of a semiconductor varies with change of doping and change of temperature. All these fundamental concepts of charge transport form the basis to understand the behaviour solid state devices in later chapters.

2.2   Electron Energy States of an Isolated Atom

The four quantum numbers required to specify the electron state in an atom are as follows :

1. The principal quantum number 'n' which determines the size and the energy of electron orbit. It can have only integral values n = 1, 2, 3, 4, ... and the corresponding electron orbits are denoted by K, L, M, N respectively.

   The number of electrons that any orbit can accommodate is $2n^2$.

An orbital quantum number 'l' that determines the shape of an electron orbit. It can have values $l = 0, 1, 2, ... (n - 1)$ and the corresponding subshells are denoted by s, p, d, f respectively.

The number of electrons contained in a subshell are $2(2l + 1)$.

An orbital magnetic quantum number m_l which determines the orientation of an orbit in space. It can have values form $+l$ to $-l$ including zero. For a given l, m_l has $(2l + 1)$ values.

The magnetic spin quantum number m_s that gives the direction of spin or self rotation. It can have values $+\frac{1}{2}$ and $-\frac{1}{2}$.

These four quantum numbers n, l, m_l, m_s define completely the state of an electron in an atom. Different physical and chemical properties of various elements are due to different configuration of electrons in their atoms.

As per Pauli's exclusion principle, no two electrons in an atom can have the same set of quantum numbers n, l, m_l, m_s. Hence an energy level cannot have more than two electrons. Thus the energy level occupied by an electron is discrete in an atom.

3 Formation of Energy Bands in Solids (Band Theory of Solids)

A single isolated atom has discrete energy levels. But if two atoms come closer to each other, they interact and significant changes in their energy levels can be observed.

A solid, is an aggregate of atoms in very close proximity. To form a solid, a large number of atoms are to be brought very close to each other.

Every atom is affected by the presence of the neighbouring atoms. During the formation of a solid, it is found that the energy levels of the inner shell electrons are not much affected by the presence of neighbouring atoms.

However, the energy levels of outer shell electrons are changed considerably. These electrons are shared by more than one atoms in the crystal.

As per Pauli's exclusion principle, not more than two interacting electrons can have the same energy level and therefore new energy levels must be established; which are discrete but infinitesimally different. *This group of discrete but closely spaced energy levels is called an energy band.*

Thus in solids, the allowed energy levels of an atom are modified by the proximity of other atoms. Every discrete level of an individual atom gives rise to a band in solid. Each band contains as many discrete levels as the number of atoms in the solid, i.e. in a solid containing N atoms, there are Ñ possible levels in each band and they can be occupied by 2N electrons.

①

Fig. 2.3.1(a) shows the energy level splitting in a solid as a function of interatomic distance.
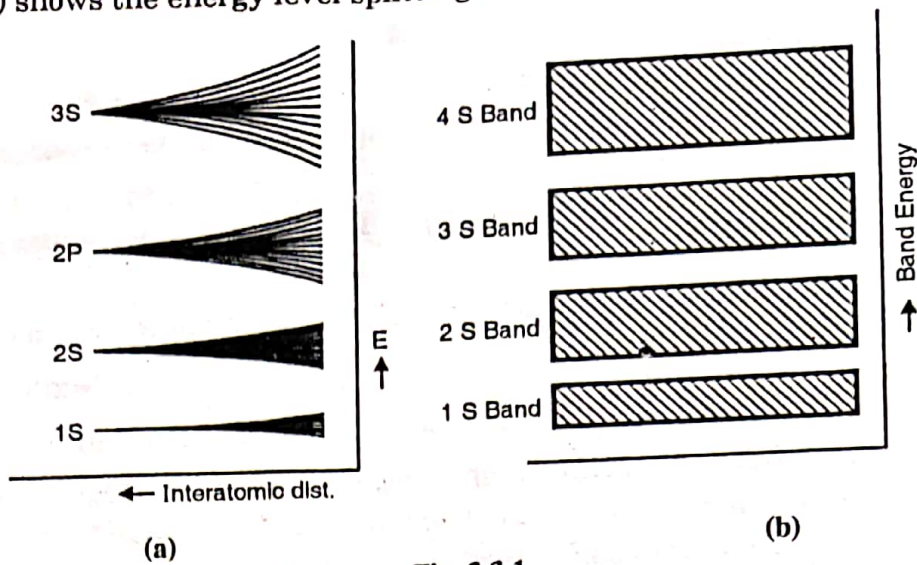

(a)

(b)

Fig. 2.3.1

The splitting of energy levels is the greatest for the outermost electrons and least for inner electrons. The individual energies in the band are so close together that each band can be considered as continuous. Fig. 2.3.1(b) shows the bands formed. The details variation of energy band with interatomic distance is shown in Fig. 2.3.2 for group IV elements.
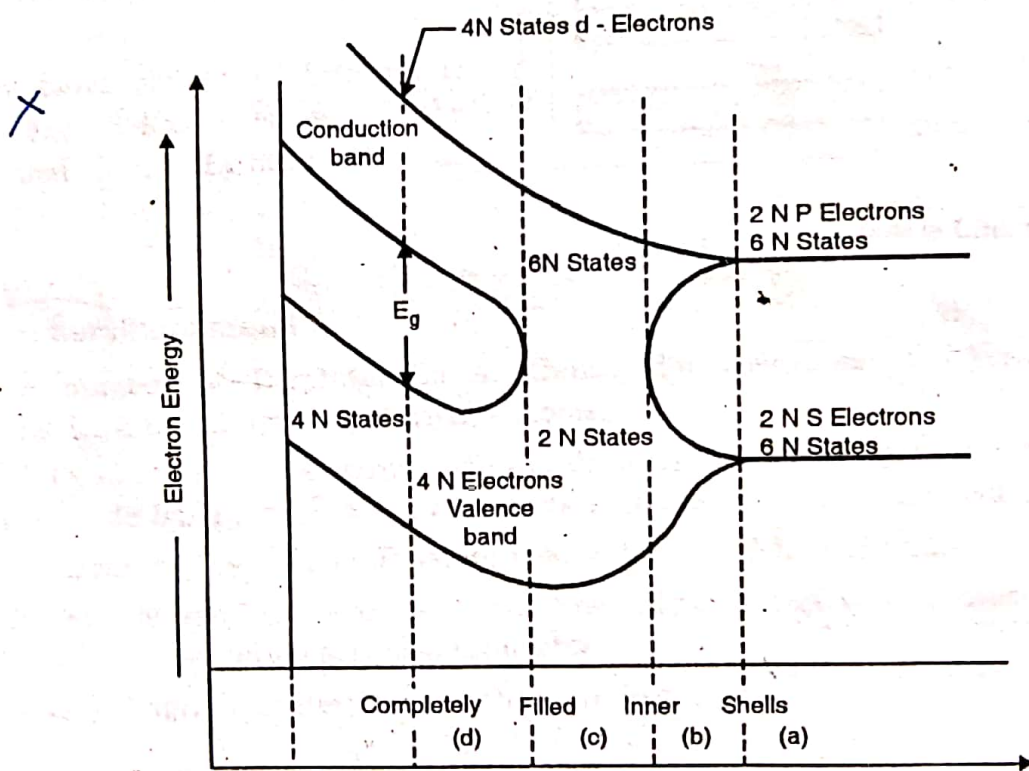

Fig. 2.3.2 : Energy band diagram showing variation of electron energy with interatomic spacing for Group IV elements

Energy Band Structures of Certain Solids

➤ [S-01, W-02, W-06]

The formation of energy bands in solids can be explained for some typical solids as follows :

Energy bands in Lithium metal

The atomic number of Lithium is 3. Hence Lithium atom has 3 electrons ($1S^2$ $2S^1$). Consider that Lithium solid has been formed from N Lithium atoms. For these N atoms, there will be 3N electrons. 2N electrons will occupy N different energy levels in K shell i.e. 1S band. This energy band will be completely filled inner band.

The remaining N electrons will occupy the higher energy L shell, i.e. 2S band with N energy levels and capacity of 2N electrons. Thus the 2S band of Lithium is only half filled and all other higher bands are completely empty.

The energy band picture of Lithium is shown in Fig. 2.4.1. Because of the partially filled band, Lithium acts as conductor.
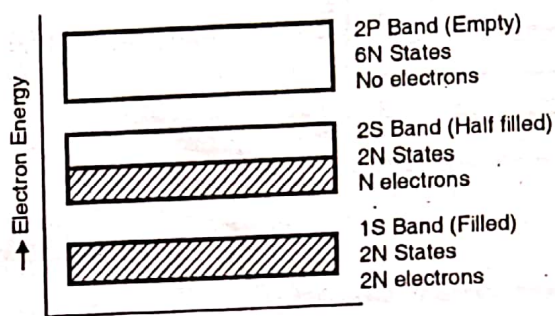
2P Band (Empty)
6N States
No electrons

2S Band (Half filled)
2N States
N electrons

1S Band (Filled)
2N States
2N electrons

Fig. 2.4.1 : Energy bands in Lithium

Energy bands in Beryllium metal

o The atomic number of Beryllium is 4. Hence the electronic configuration is $1S^2$ $2S^2$. Consider a Beryllium crystal with N atoms.

o Out of total 4N electrons, 2N electrons will fill 1S band completely and remaining 2N electrons will fill 2S band completely. Hence Beryllium seems to act as an insulator.

o However in Beryllium, the 2S and 2P bands overlap forming a hybrid band.

o The capacity of this band is 8N while it contains only 2N electrons. Because of this partially filled band, Beryllium acts as a conductor.

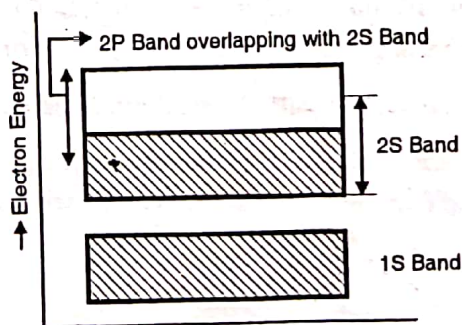o The energy band diagram for Beryllium is shown in Fig. 2.4.2.

2P Band overlapping with 2S Band

2S Band

1S Band

Fig. 2.4.2 : Energy bands in Beryllium

③

(c)   Energy bands in Diamond

o   Diamond is an allotropic form of carbon. Hence the atomic number is 6 and the electronic configuration is $1S^2 \, 2S^2 \, 2P^2$.

o   In a Diamond crystal with N atoms, the $6N$ electrons will be distributed as 2N electrons in 1S band, 2N electrons in 2S band and 2N electrons in 2P band.

o   The 2P level has three subshells $2P_x$, $2P_y$ and $2P_z$.

o   All the 2N electrons will occupy the $2P_x$ band while $2P_y$ and $2P_z$ bands will be empty. [Hund's rules for pairing of electrons is applicable only to fill the electrons in discrete energy levels of an isolated atom. But in solids, there are energy bands instead of discrete energy levels. The band of lowest energy is filled first, then the next lowest and so on.]

o   The interatomic distance in Diamond is such that, the unfilled $2P_y$ band is separated from the filled $2P_x$ band by 6 eV energy gap.

o   No free electron is available for conduction. Hence Diamond acts as an insulator.

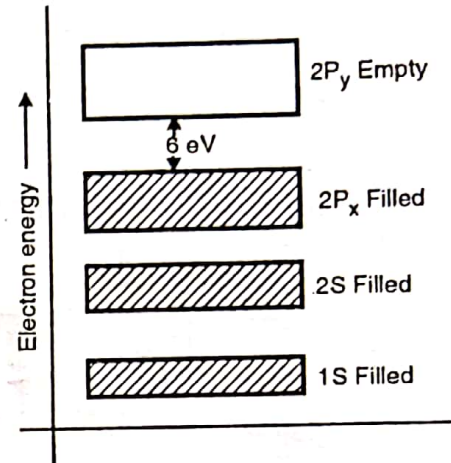o   The energy band diagram for Diamond is shown in Fig. 2.4.3.

Fig. 2.4.3 : Energy bands in Diamond

(d)   Energy bands in Silicon

o   Silicon has atomic number 14. Hence the electronic configuration is $1S^2 \, 2S^2 \, 2P^6 \, 3S^2 \, 3P^2$.

o   For a silicon crystal with N atoms, the 14 N electrons will be distributed as follows :

2N electrons in 1S band

2N electrons in 2S band

6N electrons in 2P band

2N electrons in 3S band

2N electrons in $3P_x$ band

o   Thus $3P_x$ band is completely filled and the unfilled $3P_y$ band is separated from it by, an energy gap 1.1 eV. All bands above $3P_y$ are empty.

o   The energy band diagram of Silicon is shown in Fig. 2.4.4.

o   With enough thermal energy some electrons in $3P_x$ band can overcome the gap of 1.1 eV and occupy the empty states in $3P_y$ band. Thus Silicon acts as a semiconductor.
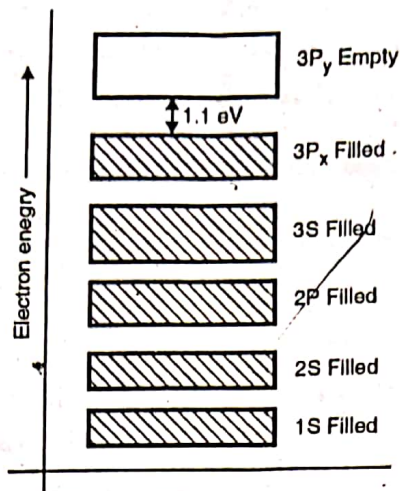
Fig. 2.4.4 : Energy bands in Silicon

Valence Band, Conduction Band and Forbidden Energy Gap

➤ [S-06]

In an atom, the electrons in the inner shells are tightly bound to the nucleus while the electrons in the outermost shell (i.e. the valence electrons) are loosely bound to the nucleus.

During the formation of a solid, a large number of atoms are brought very close together, the energy levels of these valence electrons are affected most.

The energies of inner shell electrons are not affected much.

The band formed by a series of energy levels containing the valence electrons is called the **Valence Band.**

It is the highest occupied energy band. It may be completely filled or partially filled with electrons.

The next higher permitted energy band is called the **Conduction Band.**

It may also be defined as the lowest unfilled permitted energy band.

It may be empty or partially filled with electrons.

The electrons can move freely in the conduction band and hence the electrons in conduction band are known as conduction electrons.

The energy gap between the valence band and conduction band is called the **Forbidden Energy Gap** or the **Forbidden Band.**

This band is formed by a series of non-permitted energy levels above the top of valence band and below the bottom of conduction band.

This energy gap is denoted by E_g and is the amount of energy to be supplied to the electron in valence band to get excited into the conduction band.

The band diagram is as shown in Fig. 2.5.1(a).

⑤

- This makes the outermost shells of all atoms stable. Therefore not a single free electron is available at the absolute zero temperature i.e. at 0° K or – 273° C.

- Hence the intrinsic semiconductor acts like an insulator at the absolute zero temperature.

Why are the Si and Ge referred to as tetravalent atoms ?

- As both Silicon and Germanium atoms have four electrons in their valence shells they are referred to as tetravalent atoms.

- We can apply the same logic to define "trivalent" and "pentavalent" atoms.

- Trivalent means atoms having only three valence electrons while pentavalent refers to an atom having five valence electrons.

2.12 Fermi Level and Fermi Function

> [W-02, S-03]

- A solid is formed by bringing a large number of atoms together.

- Each atomic level splits into a large number of closely packed levels and energy bands are formed.

- The filling of the bands takes place such that the lowest energy bands are filled first, then the next lowest and so on; till all the electrons are accommodated.

- In case of metals i.e. conductors, only electrons are the current carriers.

- The highest occupied energy level at 0°K is called the **Fermi Level** and the energy corresponding to it is called the **Fermi Energy E_F**.

- At 0° K, all energy states upto E_F are occupied and all states above E_F are empty.

- At higher temperatures the random thermal energy can excite the electron to higher energy states and there may be some empty states below E_F.

- Thus for conductors or metals, the highest filled energy level in the highest energy band at 0° K is the Fermi Level and its corresponding energy is E_F; the Fermi Energy.

- The distribution of electrons over a range of allowed energy levels at thermal equilibrium is given for conductors by Fermi-Dirac statistics.

- As per this, the probability that an energy state of energy E will be occupied by an electron at temperature T°K is given by,

$$P(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{KT}\right)}}$$

where, K = Boltzmann's constant

E_F = Fermi energy

$P(E)$ = Distribution function known as Fermi function.

- This function as plotted for different temperatures is shown in Fig. 2.12.1. From these curves it is observed that :

1. At $T = 0°K$, for $E < E_F$, the term $e^{\left(\frac{E-E_F}{KT}\right)} = 0$ and $P(E) = 1$

   Hence at $T = 0°K$ all energy states below E_F are certainly occupied.

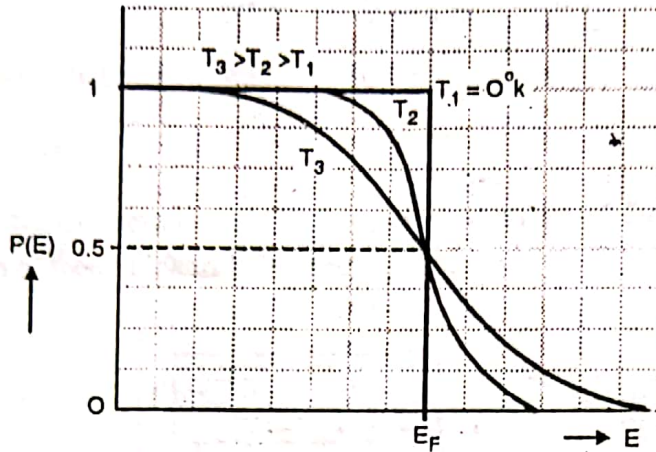**Fig. 2.12.1**

2. At $T = 0°K$ and $E > E_F$ the term $e^{\left(\frac{E-E_F}{KT}\right)} = \infty$ and $P(E) = 0$

   It means that, at $T = 0°K$ all the energy states above E_F have zero probability of occupancy and hence are empty.

3. At $T > 0°K$ and $E = E_F$, the term $e^{\left(\frac{E-E_F}{KT}\right)} = 1$ and $P(E) = \frac{1}{2}$

   i.e. the energy state at the Fermi Level has 50% probability of being occupied by an electron at any finite temperature $T > 0°K$.

2.13 Fermi Level in Semiconductors

➤ [W-03, S-03, S-04, W-05, S-06]

- In semiconductors, we have two types of current carriers i.e. electrons and holes.
- In semiconductors, Fermi Level is the reference level that gives the probability of occupancy of states in conduction band as well as the unoccupied states in valence band.
- *Fermi level in semiconductors, may be defined as the energy which corresponds to the centre of gravity of conduction electrons and holes weighted according to their energies.*

- Fermi level is only an abstraction.
- The Fermi level lies in between the conduction band and valence band.
- For an intrinsic semiconductor, the Fermi level lies at the centre of the forbidden band indicating that the states occupied in conduction band is equal to the states unoccupied in valence band.
- It means that for every electron in C.B., there is a hole in V.B.

Location of fermi energy level in semiconductors

a) Intrinsic semiconductor

o   In pure or intrinsic semiconductor the concentration of free electron in C.B. and the concentration of free holes in V.B. is exactly same.
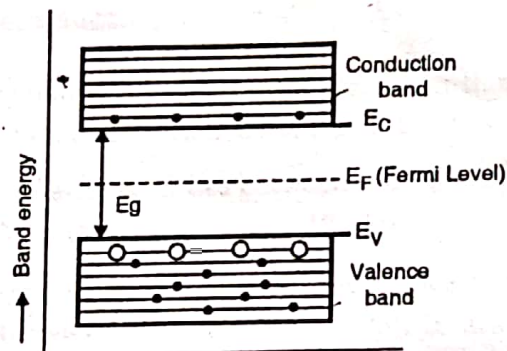

Fig. 2.13.1

o   Hence the Fermi level in intrinsic semiconductor lies exactly at the middle of the forbidden band as shown in Fig. 2.13.1.

b) Extrinsic semiconductor

o   The position of Fermi level in extrinsic semiconductor depends both on doping and on the temperature.

o   If the intrinsic semiconductor is doped, the Fermi level is always found to shift towards the energy band, that has higher concentration of charge carriers.

N-type semiconductor

o   In the N-type semiconductor the concentration of free electrons in C.B. is much greater than that of free holes in V.B. Hence the Fermi level gets shifted towards the conduction band.

o   At 0°K the Fermi energy level E_F lies between the conduction band energy E_C and the donor energy level E_D.

o   This is shown in Fig. 2 .13.2(a).

o   With the increase in temperature, the concentration of electrons and holes changes and also the position of Fermi level changes.

⑧

o　At T > 0°K the Fermi level may shift below the donor level but is always well above the centre of forbidden band.

o　Typical position of fermi level at 300°K is as shown in Fig. 2.13.2(b).
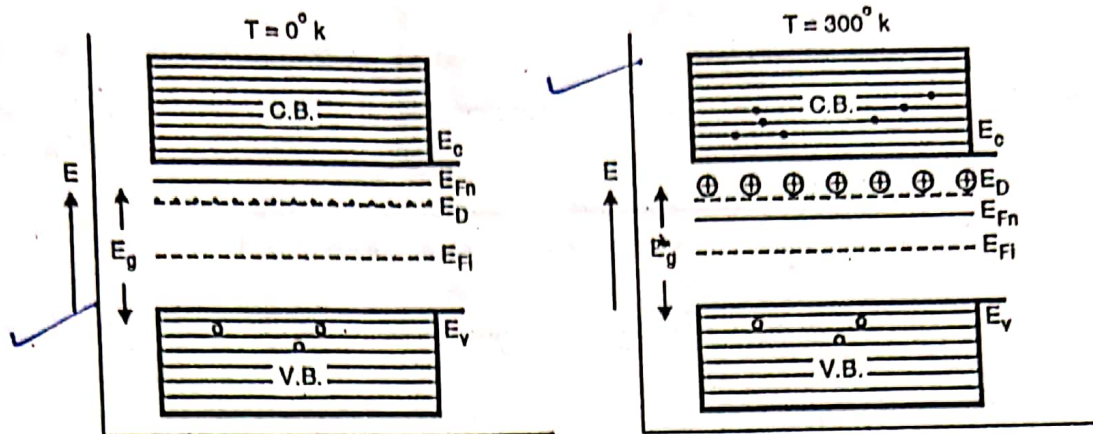
(a) T = 0°k　　　　　　　　　　　　　(b) T > 0°k

Fig. 2.13.2 : Fermi level in N-type semiconductor

2.　**p-type semiconductor**

o　In p-type semiconductor, the hole concentration in V.B. is much more than the electron concentration in C.B. Hence the Fermi level lies above the top of the valence band.

o　At 0°K the Fermi level lies between the valence band energy E_v and the acceptor energy level E_A. This is shown in Fig. 2.13.3(a).

o　As temperature rises electrons are intrinsically available in C.B. Hence Fermi level may shift above the acceptor level but is always well below the centre of forbidden band.

o　Typical position of fermi level at 300°K is as shown in Fig. 2.13.3(b).
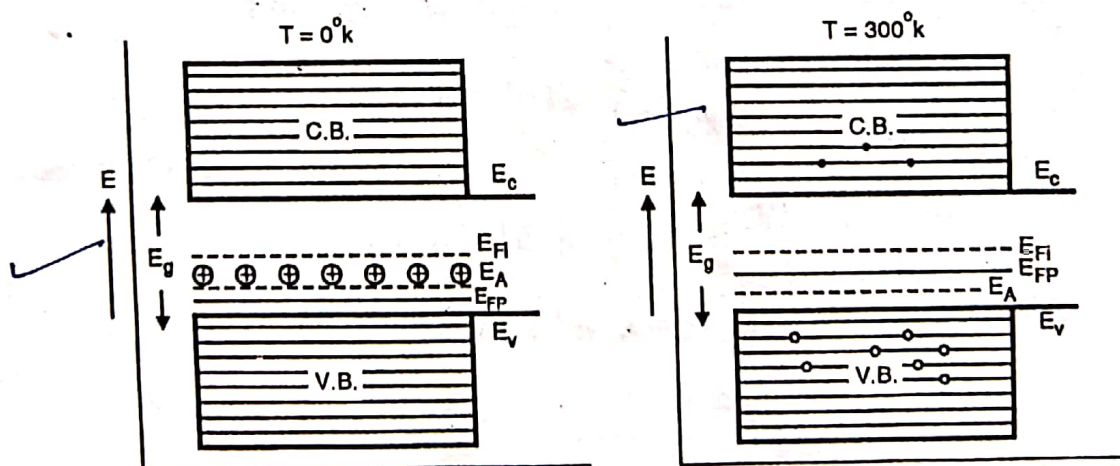
(a) T = 0°k　　　　　　　　　　　　　(b) T > 0°k

Fig. 2.13.3 : Fermi level in P-type semiconductor

2.17 Conductivity of Semiconductors

Consider a semiconductor of length l and cross-section A as shown in Fig. 2.17.1. Let a voltage V be applied across the semiconductor.

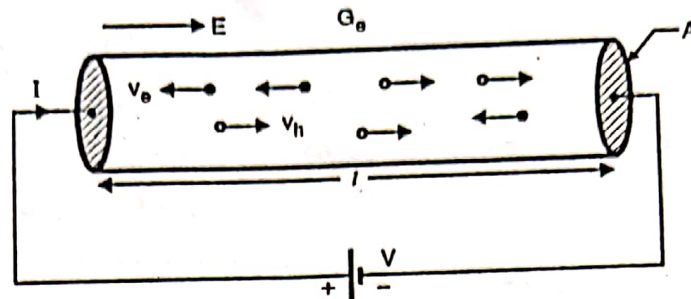The current I is due to both the current carriers i.e. electrons and holes in the semiconductor.

Fig. 2.17.1 : Semiconductor

Let n = electron density in conduction band

p = hole density in valence band

μ_e = electron mobility

μ_h = hole mobility

v_e = drift velocity of electrons

v_h = drift velocity of holes

Total current $I = I_e + I_h$ ...(2.17.1)

or $I = n \cdot e \, v_e \cdot A + p \cdot e \, v_h \cdot A$

or $I = e \cdot A (n \cdot v_e + p \cdot v_h)$ ...(2.17.2)

Now $v_e = \mu_e \cdot E = \mu_e \cdot \dfrac{V}{l}$

and $v_h = \mu_h \cdot E = \mu_h \cdot \dfrac{V}{l}$

Equation (2.17.6) gives,

$$I = e \cdot A \left[n \cdot \mu_e \cdot \frac{V}{l} + p \cdot \mu_h \cdot \frac{V}{l} \right]$$

or $I = e \cdot A \cdot \dfrac{V}{l} [n \cdot \mu_e + p \cdot \mu_h]$

Resistance $R = \dfrac{V}{I} = \dfrac{l}{e \cdot A(n \cdot \mu_e \cdot p \cdot \mu_h)} = \rho \cdot \dfrac{l}{A}$

Where $\rho \rightarrow$ Resistivity of the semiconductor

$\therefore$ Resistivity $\rho = \dfrac{1}{e(n \cdot \mu_e \cdot p \cdot \mu_h)}$

(10)

∴ Conductivity of semiconductor is $\sigma = \dfrac{1}{\rho} = e\,(n \cdot \mu_e + p \cdot \mu_h)$

or $\sigma_{a.c} = \sigma_e + \sigma_h$ ...(2.17.3)

Conductivity of intrinsic semiconductor

For the intrinsic semiconductor, the electron concentration is equal to hole concentration.

i.e. $n = p = n_i =$ intrinsic concentration of charge carriers.

∴ Conductivity of intrinsic semiconductor is

$$\sigma_i = n_i \cdot e \cdot \mu_e + n_i \cdot e \cdot \mu_h$$

or $\sigma_i = n_i \cdot e\,(\mu_e + \mu_h)$ ...(2.17.4)

Conductivity of extrinsic semiconductor

(i) n-type semiconductor

For the n-type semiconductor, $n >> p$

Therefore neglecting the hole concentration, the conductivity of n-type semiconductor can be written as,

$$\left.\begin{array}{l} \sigma_n = n \cdot e\,\mu_e \\ \text{or} \quad \sigma_n = d \cdot e\,\mu_e \end{array}\right\}$$ ...(2.17.5)

where d = concentration of donor atoms

(ii) p-type semiconductor

For the p-type semiconductor, $p >> n$

Therefore neglecting the electron concentration the conductivity of p-type semiconductor can be written as

$$\left.\begin{array}{l} \sigma_p = p \cdot e\,\mu_h \\ \text{or} \quad \sigma_p = a \cdot e\,\mu_h \end{array}\right\}$$ ...(2.17.6)

where a = Concentration of acceptor atoms

2.18 Temperature Dependence of Conductivity

- In case of metals, which are the good conductors of electricity, the resistance increases with the increase in the temperature of the conductor.

- The resistance at temperature $t°$ C is given by $R_t = R_o\,(1 + \alpha t)$.

  Where $R_o \rightarrow$ resistance at $0°$ C.

  and $\alpha \rightarrow$ temperature coefficient of resistance

- Hence the conductivity of conductors decreases with the increase of temperature.

(11)

- In case of semiconductors, the current is due to two types of carriers, the electrons and the holes. The conductivity of a semiconductor is given by,

$$\sigma_{a.c.} = e\,(n \cdot \mu_e + p \cdot \mu_h)$$

- For the intrinsic semiconductor the electrons in C.B. and holes in V.B. are equal in number at any temperature.

$$n = p = n_i = \text{intrinsic concentration of carriers}$$

- The conductivity of semiconductors depends on temperature.

- With the increase in temperature more number of electron-hole pairs are generated.

- Hence the intrinsic carrier concentration n_i also increases. Hence the conductivity of the intrinsic semiconductor also increases.

- The intrinsic carrier concentration n_i increases with temperature as per the relation.

$$\boxed{n_i^2 = A_O \cdot T^3 \, e^{\dfrac{-E_g}{KT}}}$$

     where A_O = Constant, independent of termerature

             T = Absolute temperature of the semiconductor

             E_g = Forbidden energy gap at $0°\,K$

             K = Boltzmann constant expressed in eV /°K

Thus the current through the semiconductor also increases with the increase in temperature.

2.19 The P-N Junction

➤ [W-01]

- The extrinsic semiconductors are of limited use when taken individually.

- But the situation changes considerably, when a junction of P and N-type materials is formed.

- A P-N junction is a metallurgical boundary in a semiconductor crystal in which, holes are the majority carriers on one side and electrons are the majority carriers on the other side.

- If a single piece of germanium is doped with P-type material from one side and the other half is doped with N-type material, then the plane dividing the two zones forms the P-N junction.

- Because of the difference in concentration of electrons and holes on the two sides of the junction, the electrons in N region tend to diffuse to the P region and the holes from P region tend to diffuse into N region. This process is called 'diffusion'.

- The diffused charge carriers combine at the junction to neutralize each other.

- Hence development of immobile positive ions in N region and negative ions in P-region near the junction takes place, and a charge free space called the 'depletion layer', of width of the order of few microns, is formed near the junction.

The diffusion process continues till the materials on both sides of junction equalize their Fermi levels.

The immobile ions, form parallel rows of opposite charges facing each other across the depletion layer.

Because of this charge separation, an electric potential V_B develops across the junction under equilibrium condition. This is shown in Fig. 2.19.1.

This potential is called as the 'junction potential' or the 'barrier potential'.

It prevents the further diffusion of majority charge carriers across the junction.

It should be noted that outside the barrier, on each side of the junction, the material is still neutral.
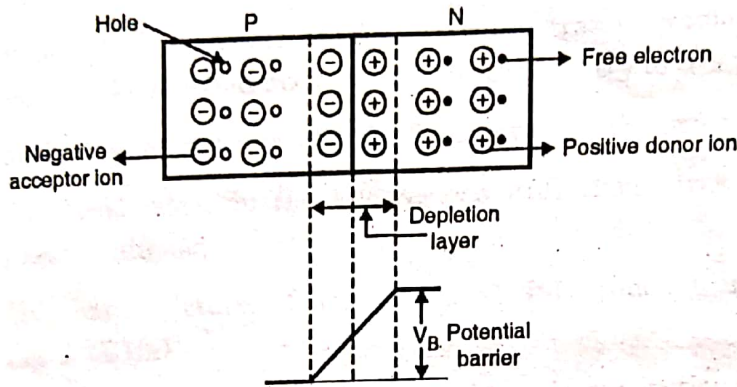

Fig. 2.19.1

20 Biasing of the P-N Junction

➤ [W-02, S-02, S-03, S-04]

Zero bias

o   When the P-N junction is in equilibrium the barrier potential does not allow the carriers to cross the junction. Hence there is no current flow across the junction.

Forward bias

o   When the P-N junction is connected in an external circuit such that, the positive terminal of battery is connected to P-region and negative terminal is connected to N-region, then the junction is said to be forward biased.

o   This is shown in Fig. 2.20.1.

o   The effect of forward bias is to reduce the potential barrier and the width of the depletion layer.

o   The holes from P-region are repelled by the positive terminal of battery towards the junction.

(13)

- A voltage of about 1 volt can permit current of about 50 mA.

- The reverse current is very small (few μA). It increases in the beginning with applied voltage and soon becomes saturated.

- When the reverse voltage increases a certain value called Zener or breakdown voltage, the reverse current suddenly and sharply increases, the curve indicating zero resistance at that point.

2.21 Energy Band Picture of a P-N Junction

> [W-02, W-03, W-04, S-05, W-06, S-07]

(a) Zero bias

- We have seen that the fermi level in the P-type material is located close to the top of the valence band while in the N-type material the fermi level is close to the bottom of the conduction band.

- When the P-N junction is formed a concentration gradient of carriers exists on the two sides of the junction due to which the carriers try to cross the junction.

- Such crossing continues till the fermi levels on the two sides of the junction equalise.

- The band edges in the two regions shift themselves so that the fermi levels are properly aligned.

- The band picture of the unbiased P-N junction at equilibrium is shown in Fig. 2.21.1(a).

- The conduction band in P region is above the conduction band in N region by an energy $e \cdot V_B$ where V_B is the potential barrier across the junction.
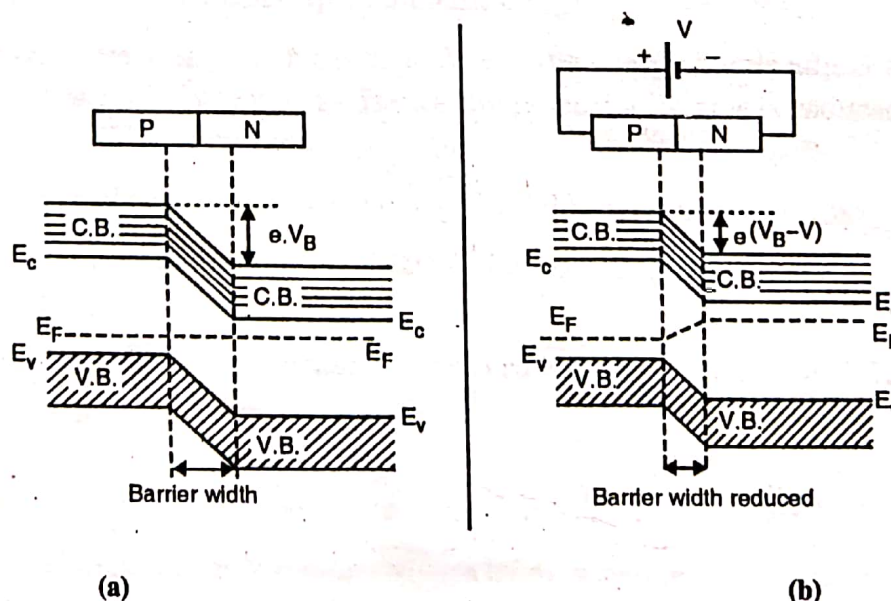

(a)   (b)

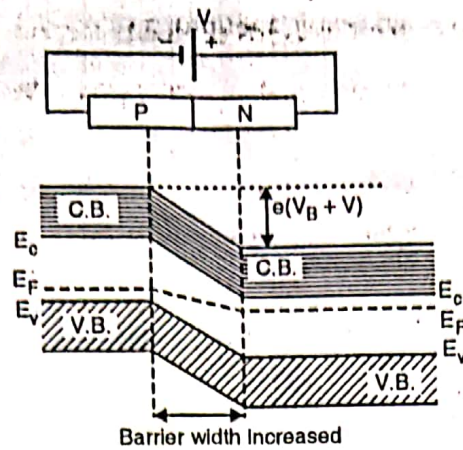Fig. 2.21.1

$e(V_B + V)$

Barrier width Increased

Fig. 2.21.1 (c)

- The majority electrons in the conduction band of N region will face the potential barrier across the junction and hence cannot cross the junction. Thus there is no current flow.

(b) Forward bias

- An external battery of voltage V is connected with its positive terminal to P region and negative to N region.

- Due to this forward bias, the equilibrium conditions are disturbed and the positions of fermi level and hence the energy bands are changed.

- As the negative terminal of external battery is connected to N- region, the energy of electrons in N region increases by an amount eV.

- Hence the fermi level on N side rises by eV and the energy bands adjust their positions so as to suit the rise of fermi level. Hence the potential barrier is reduced to e $(V_B - V)$ and the barrier width is reduced.

- The electrons in N region now face a low potential barrier and can cross it more easily.

- For the current flow to occur, the potential of external battery should be more than the barrier potential.

- The band picture of P-N junction under forward bias is shown in Fig. 2.21.1(b).

(c) Reverse bias

- The external battery of voltage V is connected with its positive terminal to Nregion.

- The energy of electrons in N region reduces by an amount eV. Hence the fermi level on N side is shifted down by eV and the potential barrier increases to e $(V_B + V)$, thereby increasing the barrier width.

- The majority electrons in N region now face a greater potential barrier in crossing the junction (But the minority carriers can easily cross it).

- The band picture of P-N junction under reverse bias is shown in Fig. 2.25(c).

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From Equations (2.24.5) and (2.24.6)

$$\Delta V_L = -\alpha' R_L \Delta I_E$$

...(2.24.7)

ΔV_L may be greater than the change in input voltage ΔV_i. Then the voltage amplification

$$A = \frac{\Delta V_L}{\Delta V_i}$$

...(2.24.8)

Will be greater than unity and the transistor acts as an amplifier. If the dynamic or incremental resistance of the emitter junction is r_e, then

$$\Delta V_i = r_e \Delta I_E$$

...(2.24.9)

From Equations (2.24.7); (2.24.8) and (2.24.9)

$$A = -\frac{\alpha' R_L \Delta I_E}{r_e \Delta I_E} = \frac{-\alpha' R_L}{r_e}$$

For example, if $r_e = 40\ \Omega$, $\alpha' = -1$, and $R_L = 3{,}000\ \Omega$, then $A = +75$.

Even though this calculation is oversimplified in essence it is correct. It gives a physical significance of why the transistor acts as an amplifier. The transistor provides power gains as well as voltage or current amplification. From the above explanation it is clear that current in the low resistance input circuit is transferred to the high – resistance output circuit. The word "transistor" which originated as a contraction of "transfer resistor" is based upon the above physical phenomenon.

2.25 Hall Effect and Hall Coefficient

➤ [S-02, W-02, W-04, S-05, W-05, S-06, W-06, S-02, W-07]

- If a metal or a semiconductor strip carrying a current I is placed in a transverse magnetic field B, an electric field E is induced in the direction perpendicular to both B and I.

- This phenomenon is known as **Hall effect.**

- It is used to detect whether a semiconductor is N-type or P-type.

- Consider a rectangular strip of a N-type semiconducting material as shown in Fig. 2.25.1.

- Under the application of some potential difference, let current I flow through it along the positive X-direction.

- Let the magnetic field B act along the positive Z direction (i.e. B acts perpendicular to plane of paper in the outward direction).

- Hence a force F acts on the charge carriers along the negative Y direction.

- As the semiconductor is N-type, the charge carriers are electrons.

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- This force due to the magnetic field has the magnitude

$$F_1 = B \cdot e \cdot v \qquad \qquad \text{...(2.25.1)}$$

where　$v \rightarrow$ drift velocity of electron

　　　　$e \rightarrow$ charge of electron

- Thus the electrons will be forced to move downward towards side 1 in Fig. 2.25.1 and side 1 becomes negatively charged with respect to side 2.
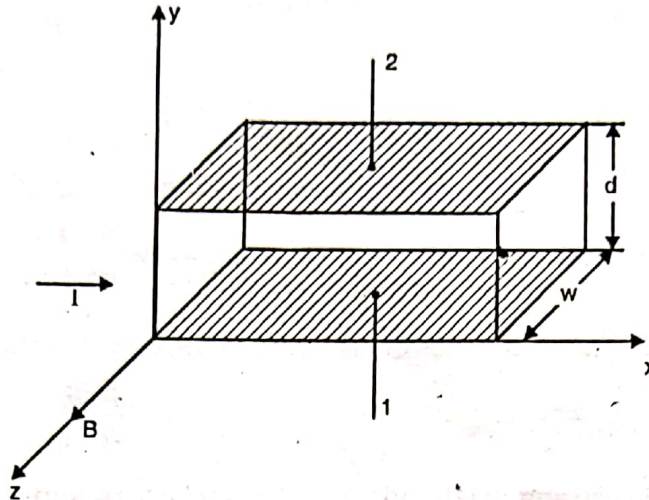

Fig. 2.25.1 : Hall effect

- As a result, an electric field is produced between the surfaces 1 and 2; and is directed along the negative Y direction.

- This field will oppose the further piling up of the electrons on surface 1.

- In the equilibrium condition, the force due to the electric field balances the force due to the magnetic field and then the electrons do not move along the Y direction.

- At this stage a steady potential difference is developed across the surfaces 1 and 2.

- This voltage is called the **Hall voltage V_H.**

- Let E be the electric field intensity due to Hall voltage V_H, then the force due to this electric field balances the force due to magnetic field.

$$\therefore e \cdot E = B \cdot e \cdot v \qquad \qquad \text{...(2.25.2)}$$
$$\therefore E = B \cdot v$$

But　$E = \dfrac{V_H}{d}$ 　　　　　　　...(2.25.3)

where d = distance between the surfaces 1 and 2

$$\therefore \frac{V_H}{d} = B \cdot v$$

or　$V_H = B \cdot v \cdot d$ 　　　　...(2.25.4)

The current $I = n \cdot e \cdot v \cdot A$

$$\therefore v = \frac{I}{n \cdot e \cdot A} \qquad \qquad \text{...(2.25.5)}$$

$$\therefore V_H = \frac{B \cdot I \cdot d}{n \cdot e \ A} = \frac{1}{n \cdot e} \cdot \frac{B \cdot I \cdot d}{A}$$

where n → Carrier concentration

A → Cross-section of the strip

Now $\dfrac{I}{A} = J =$ Current density

The Hall voltage is given by,

$$V_H = \frac{1}{n \cdot e} \cdot \frac{B \cdot I \cdot d}{A} = \frac{1}{n \cdot e} \cdot BJd \qquad \qquad ...(2.25.6)$$

The charge density ρ is the charge per unit volume of material.

$$\therefore \rho = n \cdot e$$

And area A = d · W

where W = Width of the specimen in the direction of magnetic field

$$\therefore \text{Hall voltage} \quad V_H = \frac{B \cdot I}{\rho \cdot W} \qquad \qquad ...(2.25.7)$$

If the polarity of V_H is positive at surface 2 (as in the above discussion) then the carriers must be electrons.

But if the surface 1 has positive polarity with respect to surface 2, then the carriers are positively charged holes and the semiconductor is P-type.

Thus measurement of Hall voltage helps to determine whether the given semiconductor is P-type or N-type.

We see that if V_H, B, I and W are measured, the charge density ρ can be determined.

Hence ρ = n · e can give the carrier concentration n.

The **Hall coefficient** R_H is defined as

$$R_H = \frac{1}{n \cdot e} = \frac{1}{\rho} \qquad \qquad ...(2.25.8)$$

∴ Equation (**) gives the Hall voltage
(2.25.7)

$$V_H = R_H \cdot \frac{B \cdot I}{W}$$

$$\therefore \text{Hall coefficient} \quad R_H = \frac{V_H \cdot W}{B \cdot I} \qquad \qquad ...(2.25.9)$$

$$\text{or } R_H = \frac{E \cdot d \cdot W}{B \cdot I} = \frac{E}{J \cdot B}$$

$$\text{Since } J = \frac{I}{d \cdot W}$$

Thus the Hall coefficient R_H of a semiconductor material may be defined as the electric field strength per unit surface current density due to unit transverse magnetic field.

The conductivity of the material is given as $\sigma = n \cdot e \mu$

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where $\quad \mu \rightarrow$ mobility of the carrier

$$\therefore \ \sigma = \frac{1}{R_H} \cdot \mu$$

$$\therefore \quad \boxed{\text{Mobility } \mu = R_H \cdot \sigma} \qquad \qquad \text{...}(2.25.10)$$

Thus measurement of conductivity and Hall coefficient determines the mobility of carrier.

Applications of Hall effect

The Hall effect can be used for :

1. Determination of type of semiconductor.

2. Determination of carrier concentration.

3. Determination of mobility of carriers.

2.26 Hall Angle

- The net electric field E in the specimen is a vector sum of electric field component in the x-direction because of flow of current, E_x and electric field due to Hall effect i.e. E_H as shown in Fig. 2.26.1.

- Hence, resultant electric field E acts at an angle θ_H to the x-axis and this angle is known as Hall angle.
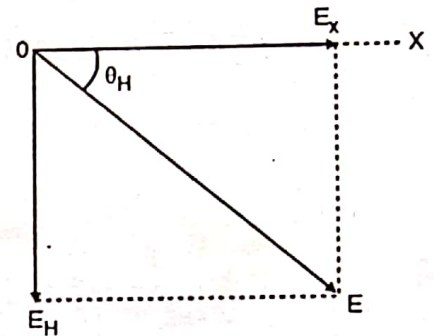

Fig. 2.26.1

- So,

$$\text{Hall angle} \quad \theta_H = \tan^{-1}\left(\frac{E_H}{E_x}\right) \qquad \qquad \text{...}(2.26.1)$$

We have, $\qquad E_H = \frac{V_H}{d}$ and

- We can write electric field component in the x-direction.

$$E_x = \frac{\text{Voltage drop along the length}}{\text{Length of specimen}}$$

$$E_x = \frac{IR}{l} = \frac{I \times \text{resistivity} \times l}{l \times A}$$

$$E_x = \frac{I}{A} \cdot \rho_0 = \frac{J}{\sigma}, \text{ where } \rho_0 \rightarrow \text{resistivity.}$$

- Put the value of E_x and E_H in Equation (30), and we have,

$$\theta = \tan^{-1}\left(\frac{E_H}{E_x}\right) = \tan^{-1}\frac{V_H/d}{J/\sigma} = \tan^{-1}\frac{BI/\rho wd}{J/\sigma}$$

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$$\theta_H = \frac{BJ/\rho}{J/\sigma} = \tan^{-1}\frac{B\sigma}{\rho}$$

$$\theta_H = \tan^{-1}(\sigma B \cdot R_H) = \tan^{-1}(\mu B)$$

Revision of Important Formulae

(i) $\sigma = n \cdot e \cdot \mu$ for conductors

(ii) $\sigma_{sc} = e(n \cdot \mu_e + p \cdot \mu_n)$ for semiconductor

(iii) $\sigma_i = n_i \cdot e(\mu_e + \mu_n)$ for intrensic semiconductor

(iv) $\sigma_N = n_d \cdot e \cdot \mu_e$ for N-type semiconductor

(v) $\sigma_p = p_a \cdot e \cdot \mu_n$ for P-type semiconductor

Hall voltage $= V_H = \dfrac{BI}{\rho w}$

Hall angle $= \theta_H = \tan^{-1}(\mu B)$

Resistivity $= R = \rho\dfrac{l}{A}$

2.27 Solved Examples

Ex. 2.27.1 : Determine the mobility of electrons in copper assuming that each atom contributes one free electron for conduction.

For copper : resistivity $= 1 \cdot 7 \times 10^{-6}$ ohm.cm. density $= 8 \cdot 96$ gm/cc

At. Wt. $= 63.5$, Avogadro number $= 6 \cdot 02 \times 10^{23}$ /gm. mole

Data : Resistivity $\rho = 1 \cdot 7 \times 10^{-6}$ Ω cm., density d $= 8.96$ gm/cc,

Avogadro no. $N = 6 \cdot 02 \times 10^{23}$ At. wt . A $= 63.5$

Formula required : $\sigma = n \cdot e \cdot \mu_e$

Soln. : Number of atoms of copper present per unit volume is

$$\cong \frac{6 \cdot 02 \times 10^{23}}{63 \cdot 5} \times 8 \cdot 96 = 8 \cdot 496 \times 10^{22}$$

As every atom contributes one free electron, the number of free electrons per unit volume, i.e. the electron density is $8 \cdot 496 \times 10^{22}$ per c.c.

∴ The conductivity is

$$\sigma = n \cdot e \cdot \mu_e$$

The mobility of electrons is

$$\mu_e \cong \frac{\sigma}{n \cdot e}$$

$$\text{or } \mu_e = \frac{1}{\rho \cdot n \cdot e \cdot} \left(\text{if } \sigma = \frac{1}{\rho}\right)$$

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$$= \frac{1}{(1.7 \times 10^{-6} \times 8.496 \times 10^{22} \times 1.6 \times 10^{-19})}$$

$$= 43.273 \text{ cm}^2/\text{volt} \cdot \text{sec.}$$

$$\therefore \sigma = 43.273 \text{ cm}^2 / \text{volt} \cdot \text{sec.}$$

Ex. 2.27.2 : A copper wire 0.1 m long and 1.7 mm² cross-section has a resistance 0.1 ohm when subjected 1 Volt potential difference between its ends. Calculate the density of free electrons in the me and the mobility of these electrons. Assume that

Density of copper = 8.96 gm/cc

Atomic weight of copper = 63.5

Avogadro number = 6.02×10^{23} / gm mole

Data :

$$\text{Length } l = 0.1 \text{ m}$$

$$\text{Cross-section } A = 1.7 \text{ mm}^2$$

$$= 1.7 \times 10^{-6} \text{ m}^2$$

$$\text{Resistance } R = 0.1 \ \Omega$$

63.5 gm. Of copper contain 6.02×10^{23} atoms

$$\text{Density of copper} = 8.96 \text{ gm/cc}$$

Formulae required

$$R = \rho \cdot \frac{l}{A}, \quad \sigma = n \cdot e \cdot \mu_e$$

Soln. : Therefore number of copper atoms per unit volume are :

$$= \frac{6.02 \times 10^{23}}{63.5} \times 8.96 = 8.49 \times 10^{22} \text{ per c.c.}$$

$$= 8.49 \times 10^{28} \text{ per m}^3$$

Each copper atom contributes one free electron for conduction.

$\therefore$ The number of free electrons per unit volume $= 8.49 \times 10^{28} / \text{m}^3$

Now resistance of copper is

$$R = \rho \cdot \frac{l}{A}$$

where $\rho \to$ resistivity of copper

$$\therefore \rho = \frac{R \times A}{l} = \frac{0.1 \times 1.7 \times 10^{-6}}{0.1}$$

$$\therefore \rho = 1.7 \times 10^{-6} \ \Omega \cdot \text{m}$$

$\therefore$ Conductivity of copper is

$$\sigma = \frac{1}{\sigma} = \frac{1}{1.7 \times 10^{-6}} \text{ mho/m}$$

Now $\sigma = n \cdot e \cdot \mu_e$

where $n \to$ electron concentration

$\mu_e \to$ electron mobility

Mobility $\mu_e = \dfrac{\sigma}{n \cdot e} = \dfrac{1}{1.7 \times 10^{-8} \times 8.49 \times 10^{28} \times 1.6 \times 10^{-19}}$

or $\mu_e = 0.0433 \times 10^{-3} \text{ m}^2 / \text{V.sec}$

2.27.3 : Find the drift velocity for an electron in silver wire of radius 1 mm and carrying a current of 2 A. Density of silver is 10.5 gm/cc.

Avogadro number $= 6.025 \times 10^{23} / \text{gm mole.}$

a :   Density of silver $= 10.5$ gm/cc.

At. wt. of silver $= 108$

mula required : $I = q \cdot n \cdot v \cdot A$

n. :

Therefore number of electrons per unit volume $= n = \dfrac{6 \times 10^{23}}{108} \times 10.5$

$\therefore n = \dfrac{6 \times 10^{23}}{10.3} = 6 \times 10^{22} \text{ per cm}^3$

or $n = 6 \times 10^{28} \text{ per m}^3$

Cross sectional area $A = \pi r^2 = \pi \times (10^{-3})^2$

$= 3 \times 10^{-6} \text{ m}^2$

$\therefore$ Now current $I = q \cdot n \cdot v \cdot A$

$\therefore v = \dfrac{I}{nqA} = \dfrac{2}{(6 \times 10^{28}) \times (1.6 \times 10^{-19}) \times (3 \times 10^{-6})}$

$= 7 \times 10^{-4} \text{ m/s.}$

2.27.4 : The resistance of copper wire of diameter 1.03 mm is 6.51 ohm per 300 m. The concentration of free electrons in copper is $8.4 \times 10^{28}/\text{m}^3$. If the current is 2 Amp., find the mobility of charge carriers and the conductivity of copper.

ta :   Diameter d $= 1.03$ mm

$= 1.03 \times 10^{-3} \text{ m}$

$\therefore$ radius r $= \dfrac{1.03}{2} \times 10^{-3} \text{ m}$

R $= 6.51 \ \Omega$

$l = 300$ mm

n $= 8.4 \times 10^{28}/\text{m}^3$

rmulae required : $R = \rho \cdot \dfrac{l}{A}, \ \mu_e = \dfrac{\sigma}{n \cdot e}$

ln. : The resistance R of a wire of length l and cross-section A is given by,

$R = \rho \cdot \dfrac{l}{A}$

here $\rho \rightarrow$ resistivity of copper

$\therefore \quad \rho = \dfrac{R \times A}{l} \qquad (A = \pi r^2)$

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$$= \frac{6.51 \times 3.14 \times \left(\frac{1.03}{2} \times 10^{-3}\right)^2}{300}$$

$$= \frac{7.228}{4} \times 10^{-6} \ \Omega \ m$$

$\therefore$ conductive $\sigma = \dfrac{1}{\rho} = \dfrac{4}{7.228} \times 10^8 = 55.34 \times 10^6$ mho/m

Now $\sigma = n \cdot e \cdot \mu_e$

where $n \rightarrow$ carrier concentration

$\mu_e =$ mobility

$\therefore$ Mobility $\mu_e = \dfrac{\sigma}{n \cdot e} = \dfrac{55.34 \times 10^6}{8.4 \times 10^{28} \times 1.6 \times 10^{-19}}$

$$= 4.117 \times 10^{-3} \ m^2 / \text{volt.sec}$$

Ex. 2.27.5 : Find the resistivity of copper if each atom of copper contributes one free electron for conduction, when the following data is given.

Data :

Atomic weight of Cu = 63.5

Density of Cu = 8.96 gm/cc

Electron mobility in Cu = 43.28 cm^2 / V.sec

Avogadro number = 6.02×10^{23} per gm mole

Formulae required : $\sigma = n \cdot e \cdot \mu_e$, $\rho = \dfrac{1}{\sigma}$

Soln. : The number of copper atoms per unit volume is

$$= \frac{N}{\text{Automic weight}} \times \text{density}$$

$$= \frac{6.03 \times 10^{23}}{63.5} \times 8.96$$

$$= 8.49 \times 10^{22} \text{ atoms}$$

Each atom of Cu contributes one free electron.

$\therefore$ Electron concentration is 8.49×10^{22} per c.c. or 8.49×10^{28} per m^3

The conductivity is $\sigma = n \cdot e \cdot \mu_e$

$\therefore$ Resistivity of copper is

$$\rho = \frac{1}{\sigma} = \frac{1}{n \cdot e \cdot \mu_e}$$

$$\therefore \quad \rho = \frac{1}{8.49 \times 10^{22} \times 1.6 \times 10^{-19} \times 43.28}$$

$$= 1.7 \times 10^{-6} \text{ ohm.cm}$$

2.27.6 : Calculate the conductivity of pure silicon at room temperature when the concentration of carriers is 1.6×10^{10} per cm³. Take $\mu_e = 1500$ cm² / volt. sec.

and $\mu_h = 500$ cm²/volt.sec. at room temperature.

Data : $\mu_e = 1500$ cm²/volt sec. $\mu_h = 500$ cm² / volt sec.

$n = p = n_i = 1.6 \times 10^{10}$ per cm³.

Formula required : $\sigma_i = n_i (\mu_e + \mu_h) \cdot e$

In. : The conductivity of pure semiconductor is given by,

$$\sigma_i = n_i (\mu_e + \mu_h) \cdot e$$

$$\therefore \quad \sigma_i = 1.6 \times 10^{10} (1500 + 500) \times 1.6 \times 10^{-19}$$

$$\text{or} \quad \sigma_i = 5.12 \times 10^{-6} \text{ mho/cm.}$$

$$\therefore \quad \sigma = 5.12 \times 10^{-6} \text{ mho/cm.}$$

2.27.7 : An n-type semi-conductor is to have a resistivity 10 Ω cm. Calculate the number of donor atoms which must be added to achieve this.

Given that $\mu_d = 500$ cm² / V.S

Data : Resistivity $\rho = 10 \ \Omega$ cm., $\mu_d = 500$ cm²/ v.s.

Formulae required : Conductivity $\sigma = \dfrac{1}{\rho}$ and $\sigma = n_d \cdot e \cdot \mu_d$

oln. :

$$n_d = \frac{\sigma}{e \cdot \mu_d} = \frac{1}{\rho \cdot \sigma \cdot \mu_d}$$

$$= \frac{1}{10 \times 1.6 \times 10^{-19} \times 500}$$

$$= 1.25 \times 10^{15} \text{ per cm}^3$$

Ex. 2.27.8 : Calculate the conductivity of specimen if a donor impurity is added to an extent of one part in 10^8 Ge atoms at room temperature ?

Data :   Avogadro number $= 6.02 \times 10^{23}$ atoms/gm.mole. At·wt. of Ge $= 72.6$,

Density of Ge $= 5.32$ gm/c.c., mobility $\mu = 3800$ cm² /v.s.

Formula required : $\sigma = n \cdot d \cdot \mu_e$

Soln. :

Concentration of Ge atoms $= \dfrac{6.02 \times 10^{23}}{72.6} \times 5.32 = 4.41 \times 10^{22}$ per cm³

As there is one donar atom per 10^8 atoms of Ge

$$n_d = \frac{4.41 \times 10^{22}}{10^8} = 4.41 \times 10^{14} \text{ per cc}$$

conductivity $\sigma = n_d \cdot \mu_e$

$$= 4.41 \times 10^{14} \times 3800 \times 1.6 \times 10^{-19} = 0.268 \text{ mho/cm.}$$

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Ex. 2.27.9 : A germanium crystal is doped with a pentavalent impurity of concentration 1 ppm. If the resistivity of doped germanium is 0.3623×10^{-3} ohm-m, find the mobility of electrons in germanium. Assume that all the impurity atoms are ionised and the density of germanium atoms is 4.42×10^{28} per m^3.

Data :

$$\text{Resistivity } \rho = 0.3623 \times 10^{-3} \ \Omega \text{ m}$$

$$\text{Ge density} = 4.42 \times 10^{28} \text{ atoms / } m^3$$

$$\text{Rate of doping} = 1 \text{ ppm} = 1 \text{ impurity atom per } 10^6 \text{ Ge atoms.}$$

Formulae required : $\sigma = \dfrac{1}{\rho} \quad \mu = \dfrac{\sigma}{n_d \cdot e}$

Soln. : Conductivity of germanium is

$$\sigma = \frac{1}{\rho} = \frac{1}{0.3623 \times 10^{-3}} \text{ mho/m}$$

$$\text{or } \sigma = 2760 \text{ mho/m}$$

The rate of impurity doping is 1 impurity atom per 10^6 Ge atoms and density of Ge atoms i $4.42 \times 10^{28} \text{ /m}^3$.

All the impurity atoms are ionised. Therefore the concentration of impurity atoms i

$$n_d = \frac{4.42 \times 10^{28}}{10^6} = 4.42 \times 10^{22} \text{ atoms/m}^3$$

$$\text{Conductivity } \sigma = n_d . e \cdot \mu$$

∴ Electron mobility is

$$\mu = \frac{\sigma}{n_d \cdot e} = \frac{2760}{4.42 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$\text{or } \mu = 0.39 \text{ m}^2 \text{ / V.sec}$$

Ex. 2.27.10 : A germanium semiconductor contains 10^{-6} % Boron and has a resistivity of 0.42 ohm. m. Calcula the density and mobility of holes in the semiconductor.

$$\text{Assume density of germanium} = 5.36 \times 10^3 \text{ kg/m}^3$$

$$\text{Atomic weight of germanium} = 72.59$$

$$\text{Avogadro number} = 6.025 \times 10^{26} \text{ / kg – mole}$$

Data :

$$\text{Resistivity} = 0.42 \ \Omega \text{ m}$$

$$\text{Boron impurity} = 10^{-6} \%$$

$$\text{Density of Ge} = 5.36 \times 10^3 \text{ kg/m}^3$$

$$\text{At. wt. of Ge} = 72.59$$

Formula required : $\text{Conductivity} = \dfrac{1}{\text{resistivity}}$

Soln. :

$$\text{Avogadro number} = 6.025 \times 10^{26} \text{ / kg. Mole.}$$

∴ 72.59 kg of Ge contains 6.025×10^{26} atoms

$$\text{Density} = \text{Mass per unit volume}$$

∴ Number of Ge atoms present per unit volume will be $= \dfrac{6.025 \times 10^{26}}{72.59} \times 5.36 \times 10^3$

$$= 0.445 \times 10^{20}$$

Rate of Boron impurity is 10^{-6} % i.e. 10^{-8}

∴ 1 Boron atom is present per 10^8 Ge atoms.

If the impurity atoms are completely ionised then, the number of holes per unit volume will

$$= \frac{0.445 \times 10^{20}}{10^8}$$

$$= 4.45 \times 10^{20}$$

∴ Holes per unit volume $= 4.45 \times 10^{20}$

Now one hole is available per acceptor Boron atom.

∴ Concentration of acceptor atoms is $a = 4.45 \times 10^{20}/m^3$

The conductivity of p-type semiconductor is $\sigma_p = a \cdot e \cdot \mu_h$

and conductivity $= \dfrac{1}{\text{resistivity}} = \dfrac{1}{0.42}$ mho/m

∴ $\dfrac{1}{0.42} = (4.45 \times 10^{20}) \times (1.6 \times 10^{-19}) \times \mu_h$

∴ Mobility of holes $= \dfrac{1}{0.42 \times 4.45 \times 10^{20} \times 1.6 \times 10^{-19}}$

$$= 0.0334 \ m^2/V.sec$$

27.11 : Calculate the current produced in a small germanium plate of area 1 cm² and of thickness 0.3 mm when a potential difference of 2V is applied across the faces. Given that the concentration of free electrons is Ge is $2 \times 10^{19}/m^3$ and mobilities of electrons and holes are 0.36 m² /v · s and 0·17 m² /v · s respectively.

$$n_i = 2 \times 10^{19}/m^3 \qquad \mu_e = 0 \cdot 36 \ m^2/v \cdot s \cdot$$

$$\mu_h = 0 \cdot 17 \ m^2/v \cdot s \cdot , \qquad \text{Area } A = 1 \times 10^{-4} \ m^2$$

Voltage $V = 2$ volts.

Length $l = 0 \cdot 3$ mm $= 3 \times 10^{-4}$ m.

ula required :　　$J = \sigma E$

$$I = \frac{\sigma_i \times V \times A}{l}$$

The current density $J = \sigma E$

Or $J = \sigma \cdot \dfrac{V}{l}$; 　Also $J = \dfrac{I}{A}$

∴ $\dfrac{I}{A} = \sigma \cdot \dfrac{V}{l}$

∴ $I = \dfrac{\sigma_i \times V \times A}{l}$

Now $\sigma_i = n_i \cdot e \cdot (\mu_e + \mu_h)$

$$= 2 \times 10^{19} \times 1 \cdot 6 \times 10^{-19} (0 \cdot 36 + 0 \cdot 17)$$

$$= 1 \cdot 696 \ \text{mho/m}.$$

∴ current $I = \dfrac{1 \cdot 696 \times 2 \times 10^{-4}}{3 \times 10^{-4}}$

or $I = 1 \cdot 13$ amp.

Ex. 2.27.12 : A specimen of pure Germanium at $300°K$ has a density of charge carriers of $2.5 \times 10^{19}/m^3$. It is doped with donor impurity atoms at the rate of one impurity atom for every 10^6 atoms of germanium. All impurity atoms may be supposed to be ionized. The density of germanium atoms is 4.2×10^{28} atoms/m^3. Find the resistivity of doped germanium if the electron mobility is 0.36 m^2/vs.

Data :   Density of charge carriers at $300°K = 2.5 \times 10^{19}/m^3$

Formulae required : $\sigma_n = N_d \cdot e \cdot \mu_e$ $\rho_n = \dfrac{1}{\sigma_n}$

Soln. :

Intrinsic carrier concentration $= 2.5 \times 10^{10}/m^3$

Density of added impurity atoms i.e. $N_d = \dfrac{4.2 \times 10^{28}}{10^6}$

$= 4.2 \times 10^{22}$ atoms/m^3

The donor concentration is very large as compared to intrinsic carrier concentration and hence the intrinsic concentration can be neglected.

The conductivity of doped material is therefore

$$\sigma_n = N_d \cdot e \cdot \mu_e = 4.2 \times 10^{22} \times 1.6 \times 10^{-19} \times 0.36$$

$$= 2.492 \times 10^3 \text{ mho/m}$$

Hence the resistivity is

$$\rho_n = \frac{1}{\sigma_n} = \frac{1}{2.4192 \times 10^3} \text{ ohm m.}$$

or $\rho_n = 0.4133 \times 10^{-3} \ \Omega \text{ m.}$

Ex. 2.27.13 : Calculate the current produced in a germanium sample of area 2 sq.cm and thickness 0.1 mm when a potential difference of 4 Volts is applied across it.

Given : $n_i = 1 \times 10^{19}/m^3$

         $\mu_e = 0.36$ m^2/volt.sec

         $\mu_h = 0.17$ m^2 / volt.sec

Data. :   Area $A = 2$ cm$^2 = 2 \times 10^{-4}$ m^2

     Thickness $t = 0.1$mm $= 10^{-4}$ m

          $V = 4$ volts

Formula required : $\sigma_i = n_i \cdot e \cdot (\mu_e + \mu_h)$

Soln. : The conductivity is

$$\sigma_i = n_i \cdot e \cdot (\mu_e + \mu_h) = 1 \times 10^{19} \times 1.6 \times 10^{-19}(0.36 + 0.17)$$

$$= 1.6 \times 0.53$$

$$= 0.848 \text{ mho/m}$$

$\therefore$ current density $J = \sigma_i \times E = \sigma_i \times \dfrac{V}{t}$

$$\therefore J = 0.848 \times \frac{4}{10^{-4}} \text{ Amp / m}^2$$

$\therefore$ Current produce is $I = J \times A$

$$\therefore \quad I = 0.848 \times \frac{4}{10^{-4}} \times 10^{-4} \times 2$$

or $I = 6.784$ Amp.

27.14 : Calculate conductivity of pure silicon at room temperature if concentration of carriers is 1.6×10^{10} per cm^3. Given that mobilities of electron and hole are 1500 cm^2 /V.s and 500 cm^2/ V.s. respectively.

$\mu_h = 500$ cm^2 / V.s.

$\mu_e = 1500$ cm^2 / V.s.

$n_i = 1.6 \times 10^{10}$/cm^3

mula required : $\sigma_i = n_i e (\mu_h + \mu_e)$

n. :

We have intrinsic conductivity expression.

$$\sigma_i = n_i e (\mu_e + \mu_h)$$

$$\sigma_i = 1.6 \times 10^{10} \times 1.6 \times 10^{-19} (1500 + 500)$$

$$\sigma_i = 5.12 \times 10^{-6} \text{ mho / cm}$$

2.27.15 : A silver wire is in the form of a ribbon 0.50 cm. wide and 0.10 mm. thick. When a current of 2 amp. passes through the ribbon, perpendicular to 0.80 Tesla magnetic field calculate the Hall voltage produced. The density of silver = 10 · 5 gm/cc.

Data :   B = 0.8 Tesla, Density = 10.5 gm/cc

Formula required : $V_H = B.v.d.$ $v = \dfrac{I}{nAq}$

n. : The number of electrons in 1cc of silver are :

$$n = 6 \cdot 025 \times 10^{23} \times \frac{10 \cdot 5}{108} \approx 6 \times 10^{22} \text{ per c.c.}$$

As each silver atom contributes one electron, the number of electrons per $m^3 = 6 \times 10^{28}$

Area $A = 0.5 \times 10^{-2} \times 0.1 \times 10^{-3} = 5 \times 10^{-7}$ m^2.

$\therefore$ Hall voltage $V_H = B.v.d.$

the drift vel $v = \dfrac{I}{nAq}$

$$\therefore \quad V_H = \frac{I}{n \cdot q} \cdot \frac{B \cdot I \cdot d}{A}$$

$$= \frac{1}{6 \times 10^{28} \times 1 \cdot 6 \times 10^{-19}} \times \frac{0 \cdot 8 \times 2 \times 0.1 \times 10^{-3}}{5 \times 10^{-7}}$$

$$= 0.333 \times 10^{-7} \text{ volts}$$

2.27.16 : A copper specimen having length 1 metre, width 1 cm and thickness 1 mm is conducting 1 amp. current along its length and is applied with a magnetic field of 1 Tesla along its thickness. It experiences Hall effect and a hall voltage of 0.074 microvolts appears along its width. Calculate the Hall coefficient and the mobility of electrons in copper.

(Conductivity of copper is $\sigma = 5.8 \times 10^7$ $(\Omega m)^{-1}$]

a : $l = 1$ m, d = 1cm = 10^{-2} m

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$$W = 1 \text{ mm} = 10^{-3} \text{ m}, \quad I = 1 \text{ Amp}$$
$$B = 1 \text{ Tesla}, \quad V_H = 0.074 \times 10^{-6} \text{ Volts}$$
$$\sigma = 5.8 \times 10^7 \text{ mho/m}$$

Formulae required :

$$V_H = \frac{1}{n_q} \cdot \frac{B \cdot I \cdot d}{A}$$

Soln. :

The Hall voltage is

$$V_{II} = \left(\frac{1}{n_q}\right) \cdot \frac{B \cdot I \cdot d}{A}$$

$$\text{or} \quad V_{II} = R_H \times \frac{B \cdot I \cdot d}{A}$$

$$\therefore R_{II} = \frac{V_{II} \times A}{B \cdot I \cdot D}$$

$$= \frac{0.074 \times 10^{-6} \times (10^{-2} \times 10^{-3})}{1 \times 1 \times 10^{-2}}$$

$$\therefore R_{II} = 7.4 \times 10^{-11} \text{ m}^3/\text{c}$$

$$\therefore \text{Mobility} \quad \mu = \sigma \cdot R_{II}$$

$$= 5.8 \times 10^7 \times 7.4 \times 10^{-11}$$

$$\therefore \mu = 4.3 \times 10^{-3} \text{ m}^2/\text{volt.sec}$$

Ex. 2.27.17 : Determine the concentration of holes in Si crystals having donor concentration of $1.4 \times 10^{24}/\text{m}^3$ when the intrinsic carrier is $1.4 \times 10^{18}/\text{m}^3$. find the ratio of electron to hole concentration.

Soln. :

Intrinsic carrier concentration

$$n_i = 1.4 \times 10^{18}/\text{m}^3$$

Donor concentration $\quad n_D = 1.4 \times 10^{24}/\text{m}^3$

Concentration of electron, $\quad n = N_D = 1.4 \times 10^{24}/\text{m}^3$

Concentration of holes, $\quad p = \dfrac{n_i^2}{n} = \dfrac{(1.4 \times 10^{18})^2}{1.4 \times 10^{24}}$

$$= 1.4 \times 10^{12}/\text{m}^3$$

Ratio of electron to hole concentration $= \dfrac{n}{p}$

$$= \frac{1.4 \times 10^{24}}{1.4 \times 10^{12}} = 1 \times 10^{12}$$

Ex. 2.27.18 : The resistivity of semiconductor material was known to 0.00912 $\Omega - $m at room temperature. The flux density in the Hall model was 0.48 Wb/m². Calculate the Hall angle for a Hall coefficient of 3.55×10^{-4} m³ / coulomb.

Soln. :

Flux density in Hall Model $\quad B = 0.48 \text{ wb/m}^2$

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